# Electrical Machines 1

Hour 8

Transformer

Introduction

- Transformer is a machine that has no moving parts but is able to transform alternating voltages and currents from high to low (step-up transformer) and vice versa (step-down transformer).

- Transformers are used extensively in all branches of electrical engineering from the large power transformer employed in the T&D network to the small transformer of an electronic amplifier

Transformer

Introduction (cont)

- A simple transformer consists of two coils wound on a closed iron core as shown:
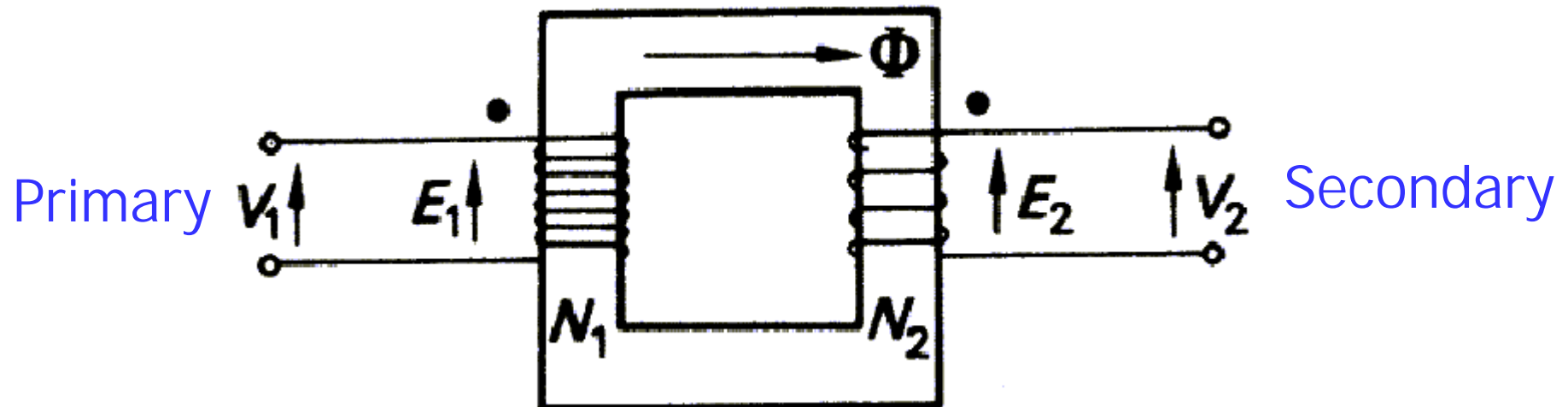
Primary

Secondary

Construction of Transformer

Basically, a transformer has two windings:

- primary winding

- Secondary winding

Each winding consists of many turns and are wound on a laminated iron core. The iron core is insulated with the windings. The core itself forms a closed iron magnetic circuit. Consequently, the windings encircle the core and the core encircles the windings.
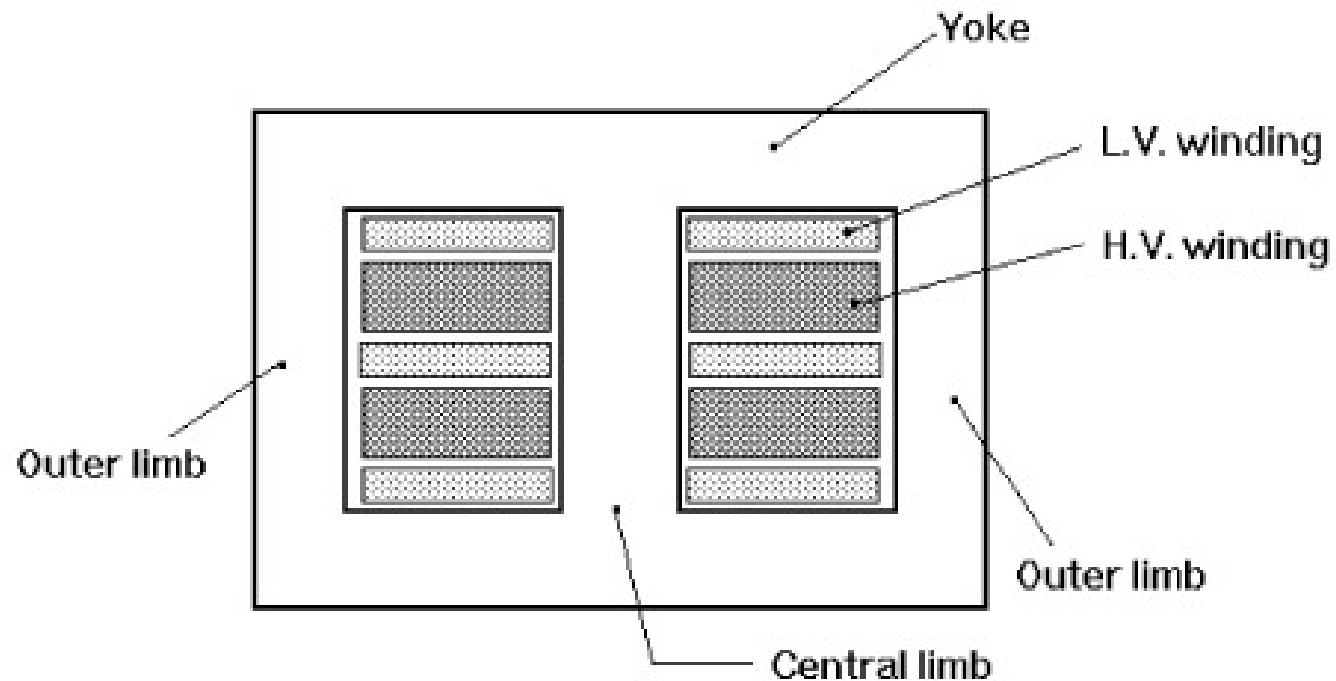
Construction of transformer

- Core type

-- windings are wound around two legs of a magnetic core.

Yoke

Limb

H.V. winding
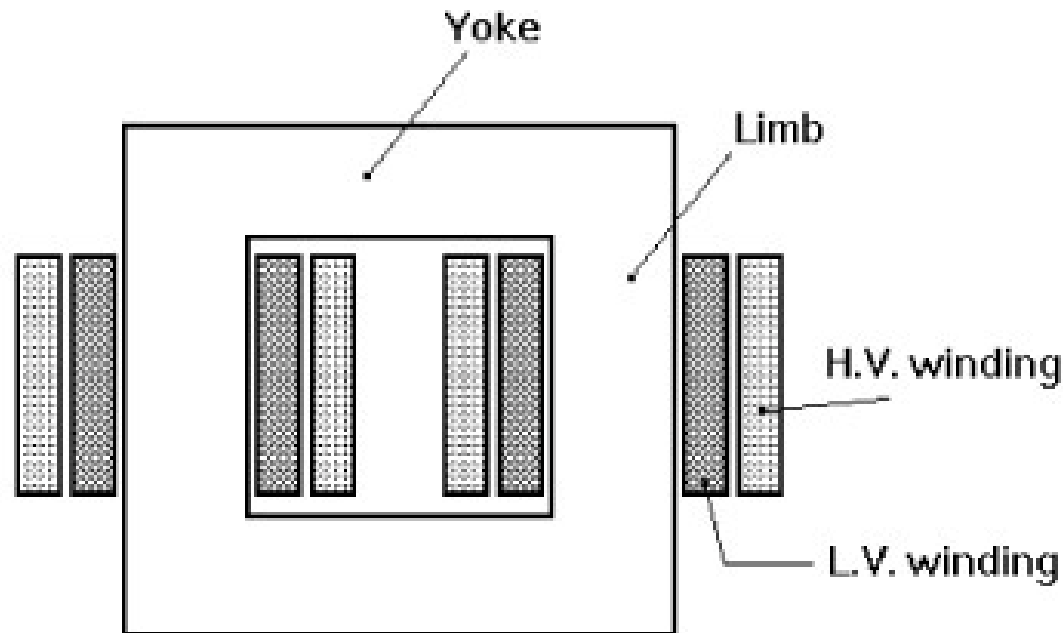
L.V. winding

Construction of transformer
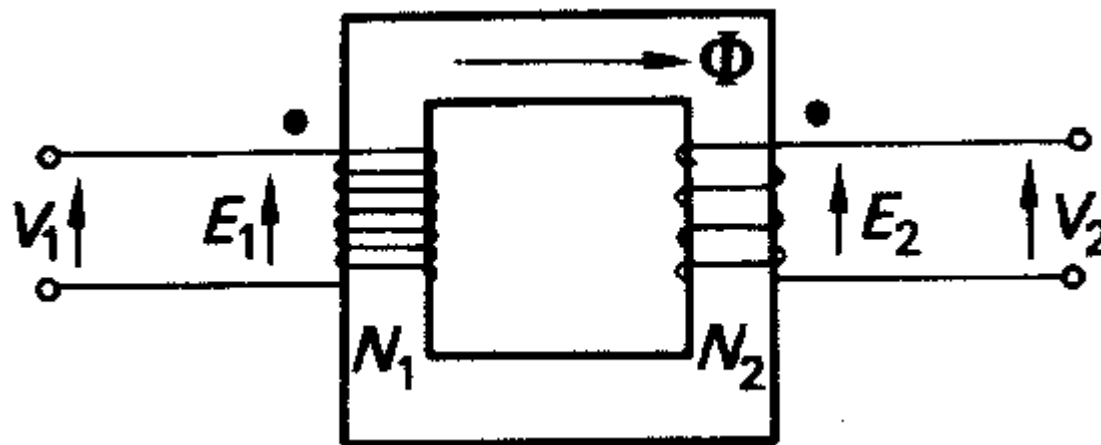
- Shell type

  -- windings are wound around the center leg of a three-legged magnetic core

Ideal Transformer

Assumptions:

1. The winding resistances are negligible (R = 0)
2. All fluxes are confined to the core and link both windings (leakage flux = 0)
3. Permeability of the core is infinite ($I_{\phi 1}$ = 0)
4. No eddy current and hysteresis losses

Ideal Transformer (cont)

When an A.C. voltage v_1 applied to the primary winding:

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt} \quad ------- (1)$$

The core flux also links the secondary winding, then:

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt} \quad ------- (2)$$

From equations (1) and (2):

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$

Ideal Transformer (cont)

- when a load is connected to the secondary winding, i_2 will flow and will provide an mmf $N_2 i_2$ for the core.

- i_1 would immediately flow to establish another mmf $N_1 i_1$ to oppose $N_2 i_2$ since no mmf is required to establish a flux in the ideal core

Then: $$N_1 i_1 - N_2 i_2 = 0$$

$$N_1 i_1 = N_2 i_2$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

Ideal Transformer (cont)

If the supply voltage is sinusoidal, then in rms values:
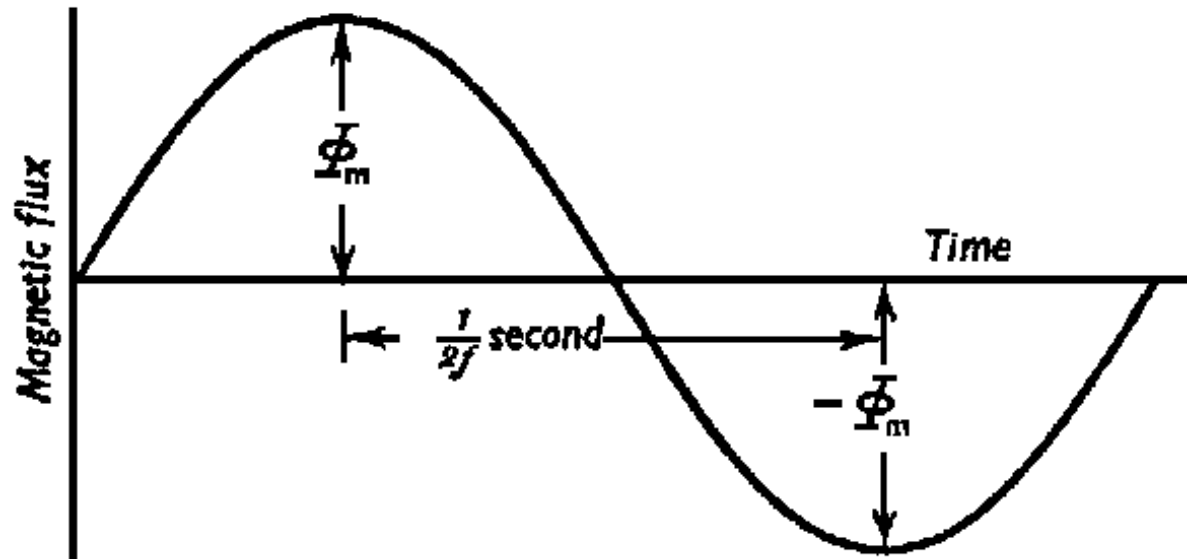
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$\boxed{V_1 I_1 = V_2 I_2}$$

$$\text{input VA} = \text{output VA}$$

Emf Equation of a Transformer (1/2)

$$\Phi = \Phi_m \sin 2\pi ft$$

instantaneous value of induced emf / turn

$$= - d\Phi/dt \text{ volts}$$

$$= - 2\pi ft \Phi_m \cos 2\pi ft \text{ volts}$$

$$= 2\pi ft \Phi_m \sin(2\pi ft - \pi/2) \text{ volts}$$

Emf Equation of a Transformer (2/2)

rms value of induced emf / turn
$$= 0.707 \times 2\pi ft\Phi_m \text{ volts}$$
$$= 4.44 \times f\Phi_m \text{ volts}$$

Hence $E_1 = 4.44N_1f\Phi_m$ volts
$E_2 = 4.44N_2f\Phi_m$ volts

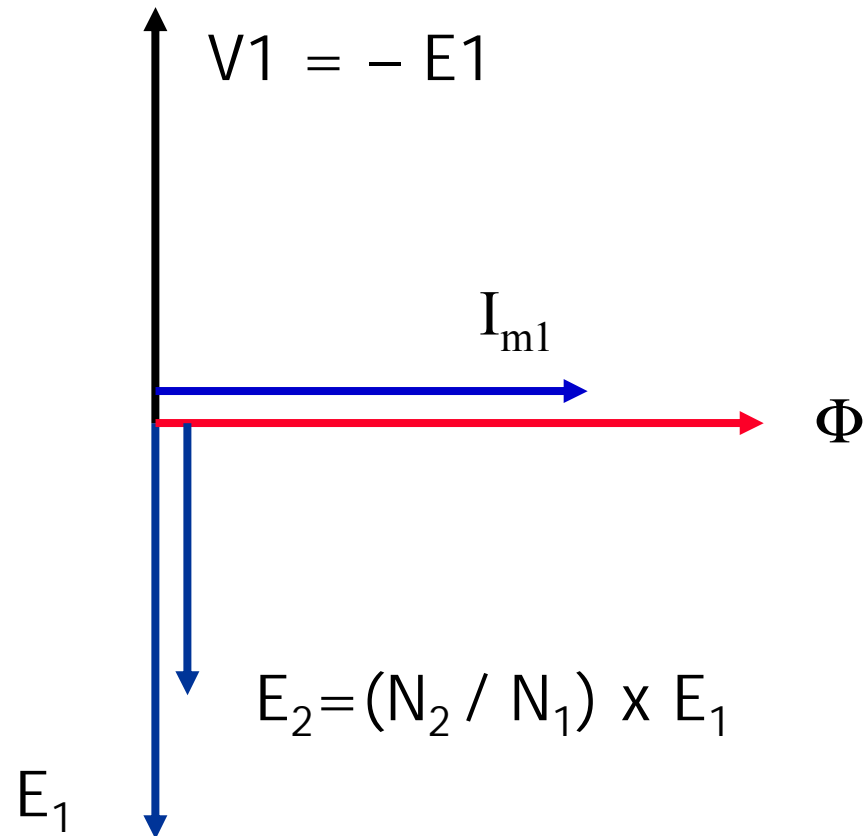$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

▶ Example

A 200kVA, 6600/400V, 50 Hz single-phase transformer has 80 turns on the secondary. Calculate: (a) the approximate values of the primary and secondary currents; (b) the approximate number of primary turns; and (c) the maximum value of the flux.

Ans: 30.3A, 500A; 1320; 0.0225Wb

Phasor Diagram of Ideal Tx under No-load

$$V1 = -E1$$

$$I_{m1}$$

$$\Phi$$

$$E_2 = (N_2 / N_1) \times E_1$$

$$E_1$$

Electrical Machines 1

Hour 9

Practical Transformer (sect 2.2)

- the windings have resistances

- not all windings link the same flux

- permeability of the core material is not infinite, and core losses occur when the core material is subjected to time-varying flux

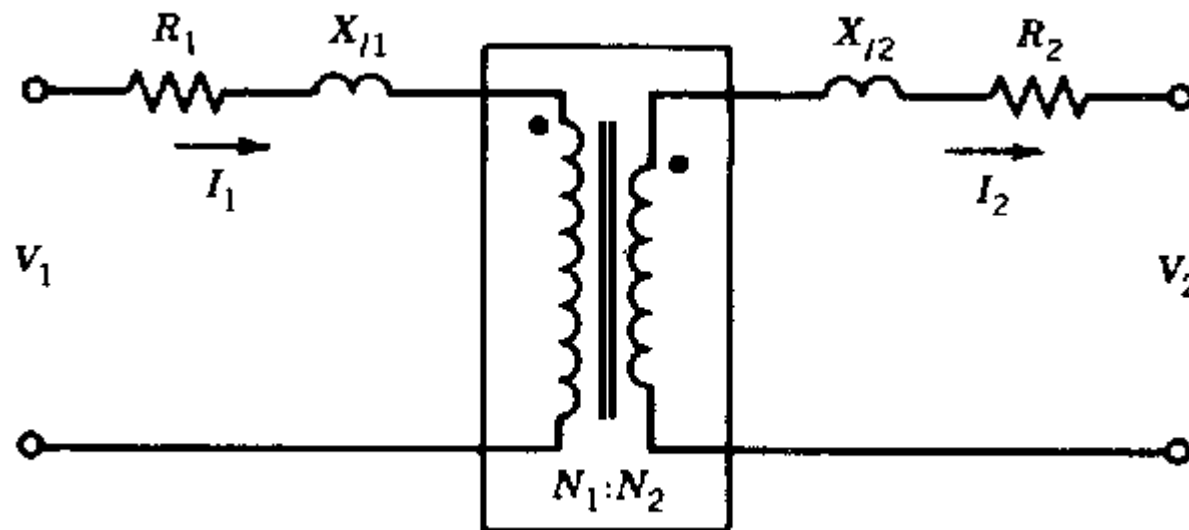# Development of the Transformer Equivalent Circuit

- Considering the winding resistances and the leakage flux
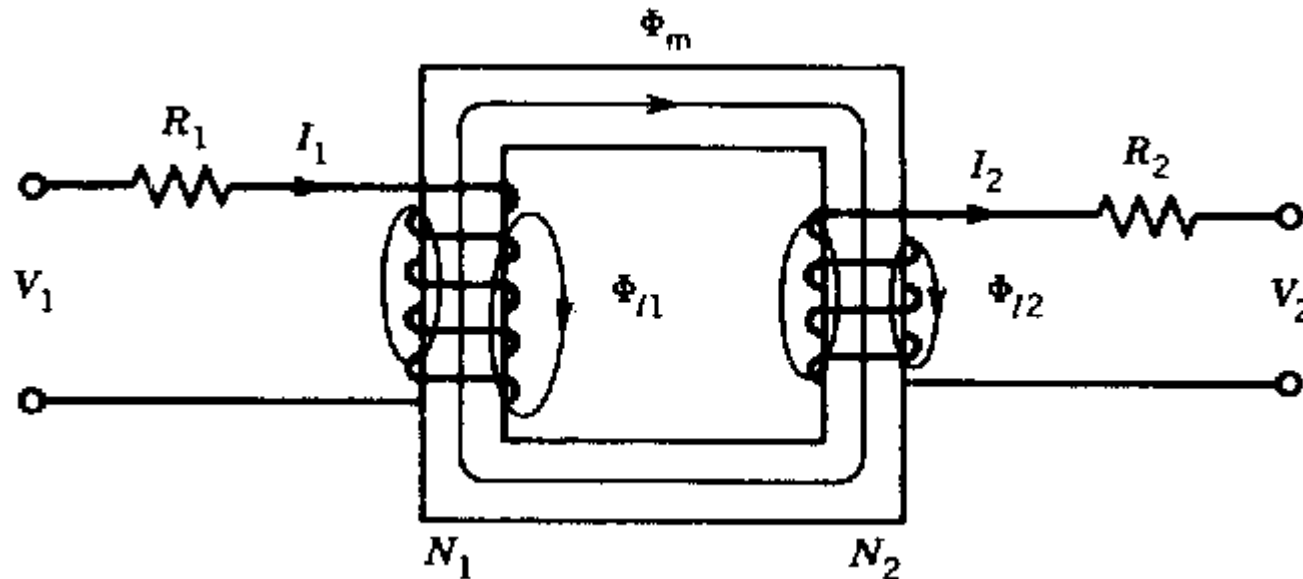
Development of the Transformer Equivalent Circuit (cont)

- Equivalent circuit obtained when the winding resistances and the leakage flux are being considered
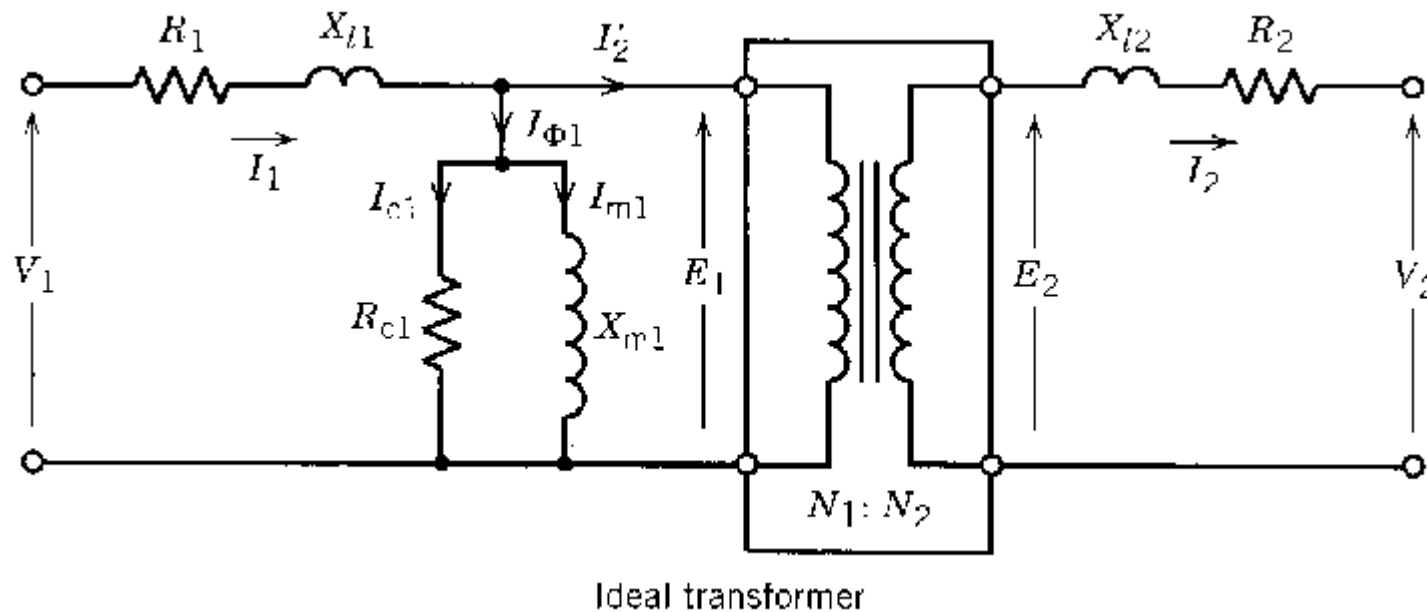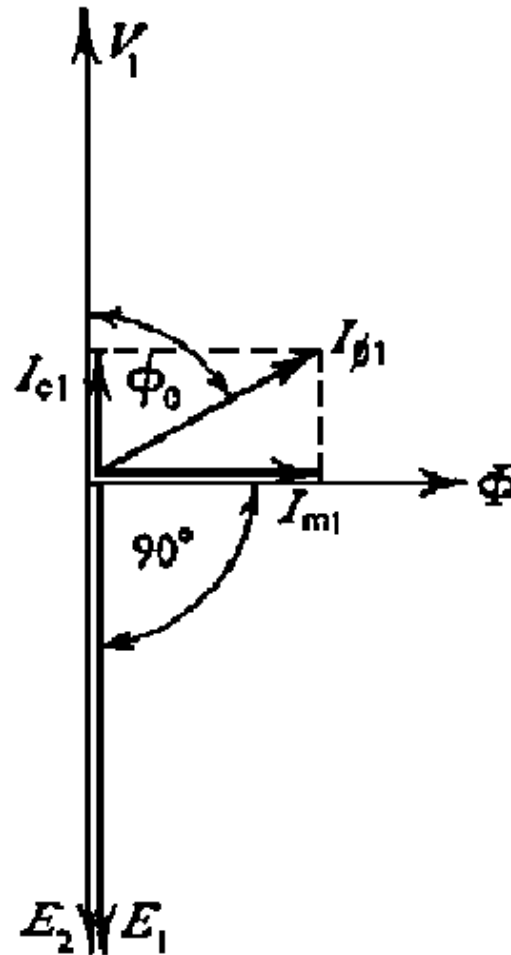

Tightly coupled transformer

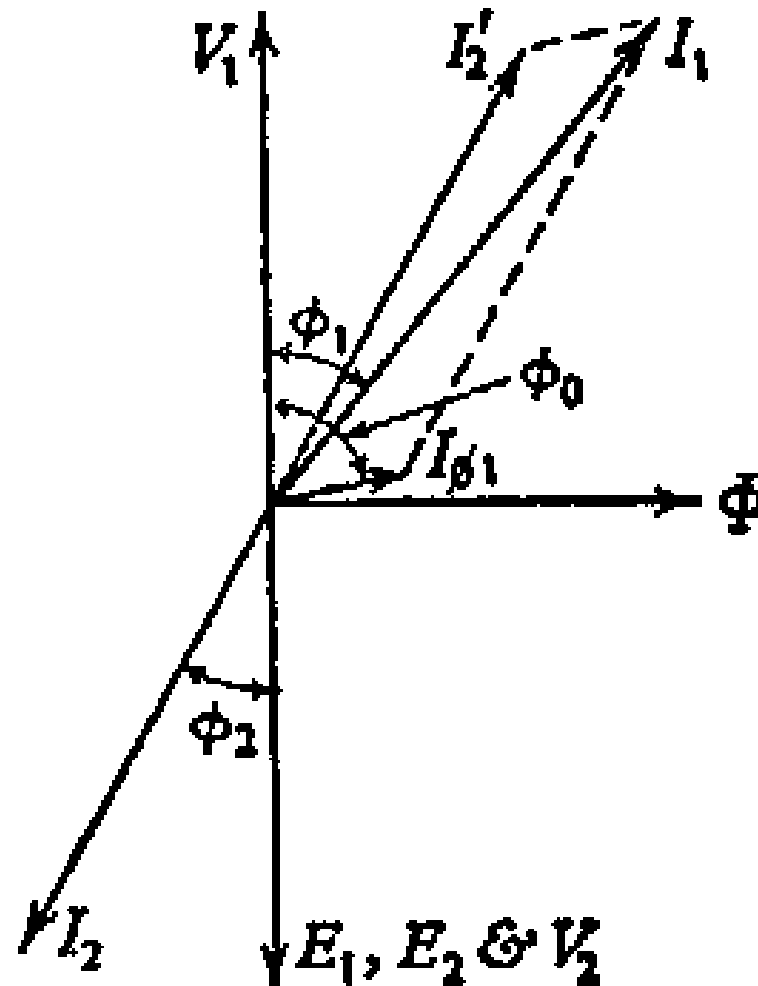# Development of the Transformer Equivalent Circuit (cont)

- Equivalent circuit obtained when the core loss and magnetizing inductance are being added

Ideal transformer

Phasor Diagram (No-load)

Phasor Diagram (On-load)

Negligible voltage
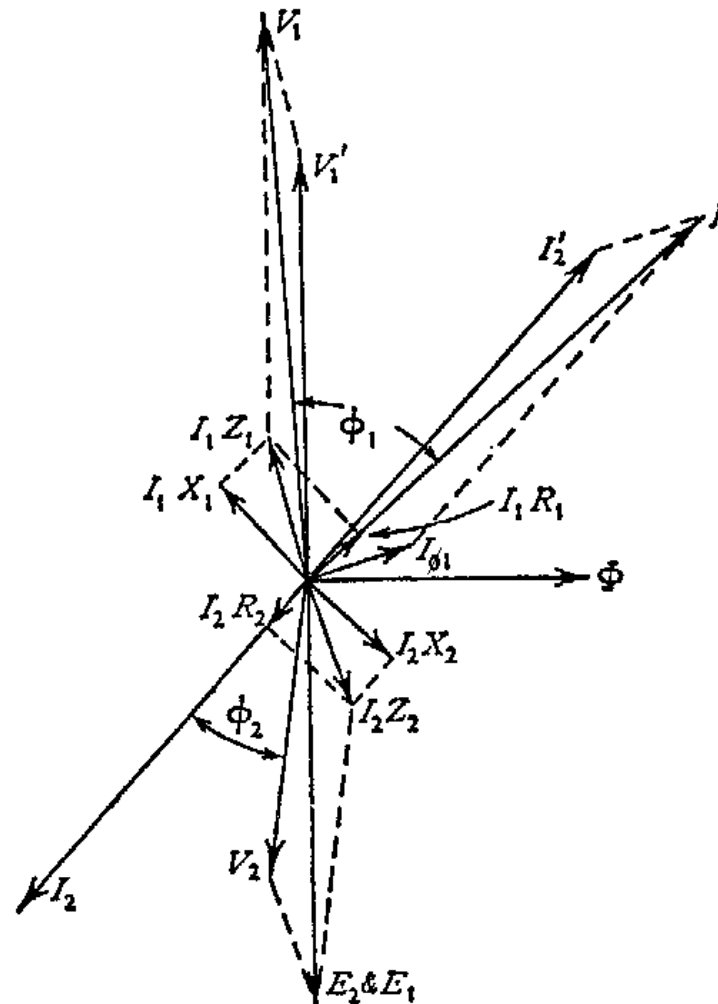drop in windings

Example

A 1-ϕ transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current is 3A at a power factor 0.2 lagging. Calculate the primary current and the power factor when the secondary current is 280A at a power factor of 0.8 lagging. Assume the voltage drop in the windings to be negligible.

Ans: I_1= 58.3A, $\cos\phi_1$=0.78 lag

Phasor Diagram (On-load)

Include voltage drop in windings

Electrical Machines 1

Hour 10

Impedance Transfer

The impedance of the secondary winding, Z_2

$$Z_2 = \frac{V_2}{I_2}$$

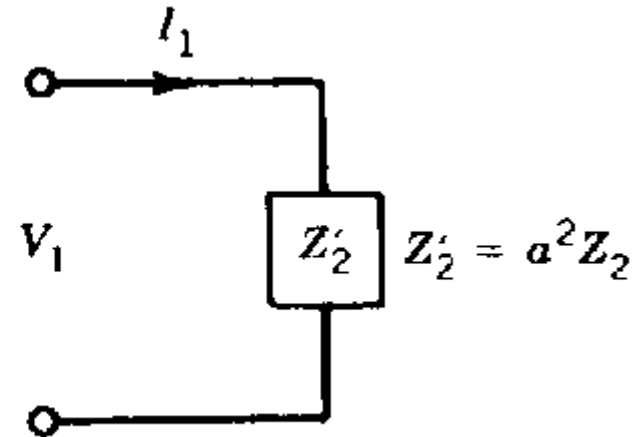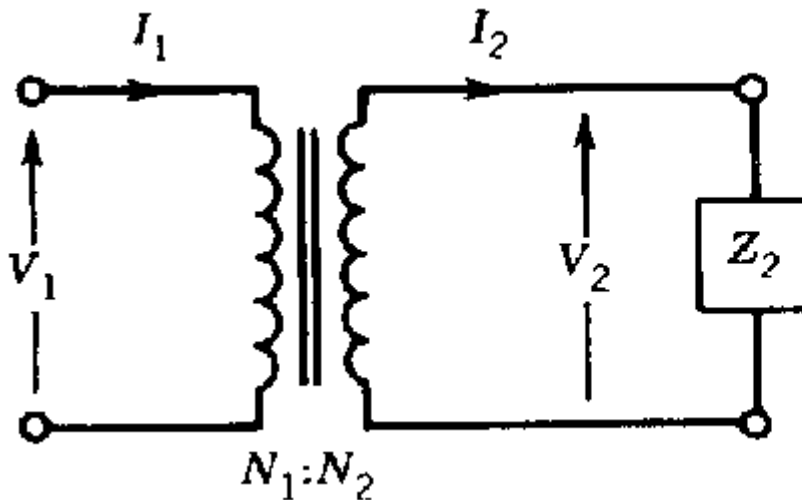The input impedance, Z_1

$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2 \frac{V_2}{I_2}$$

$$= a^2 Z_2$$

$$= Z'_2$$

Impedance Transfer (cont)

An impedance Z_2 connected in the secondary will appear as an impedance Z'_2 looking from the primary

Impedance Transfer (cont)

An impedance from the primary side can also transferred to the secondary side, and in that case its value has to be divided by the square of the turns ratio

$$Z'_1 = \frac{1}{a^2} Z_1$$

Impedance transfer is very useful because it eliminates a coupled circuit in an electrical circuit and thereby simplifies the circuit

Referred Equivalent Circuit

- Equivalent circuit when secondary resistance and reactance are referred to the primary
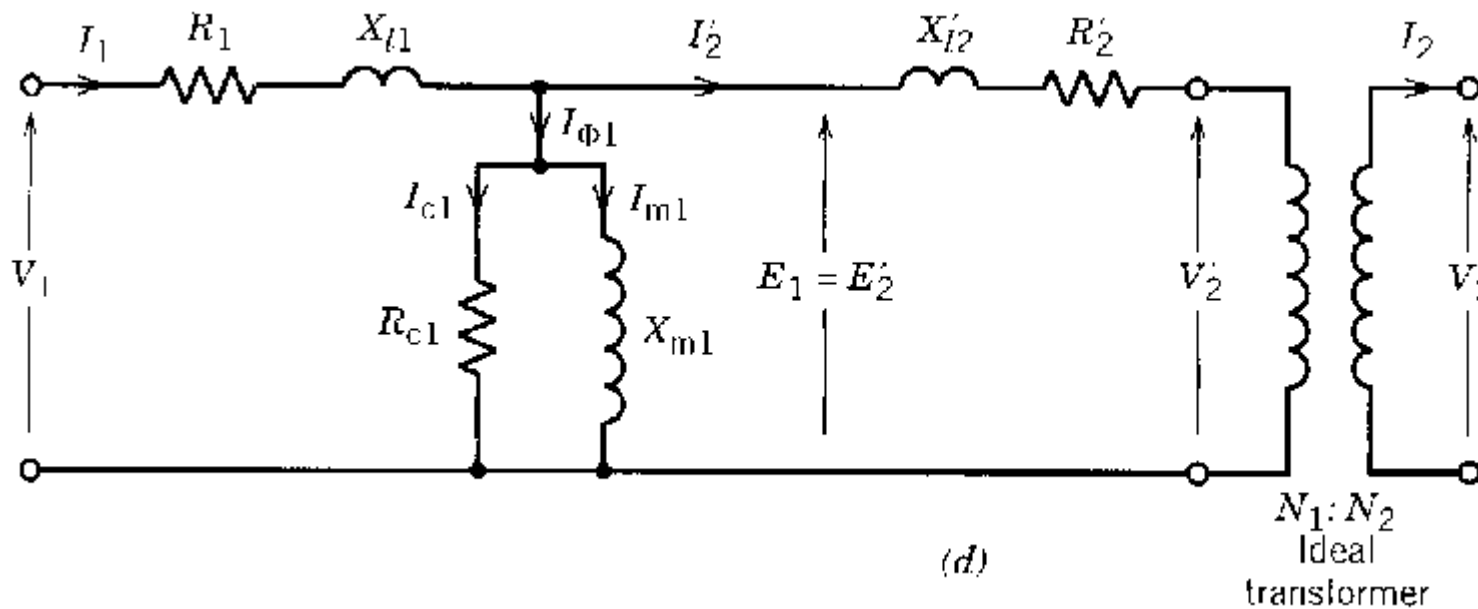
$$E_1 - E_2' = aE_2$$
$$V_2' = aV_2$$
$$I_2' = I_2/a$$
$$X_{l2}' = a^2 x_{l2}$$
$$R_2' = a^2 R_2$$

$N_1 : N_2$
Ideal transformer

(d)

Referred Equivalent Circuit (cont)

With all quantities (voltages, currents, and impedances) referred to the primary
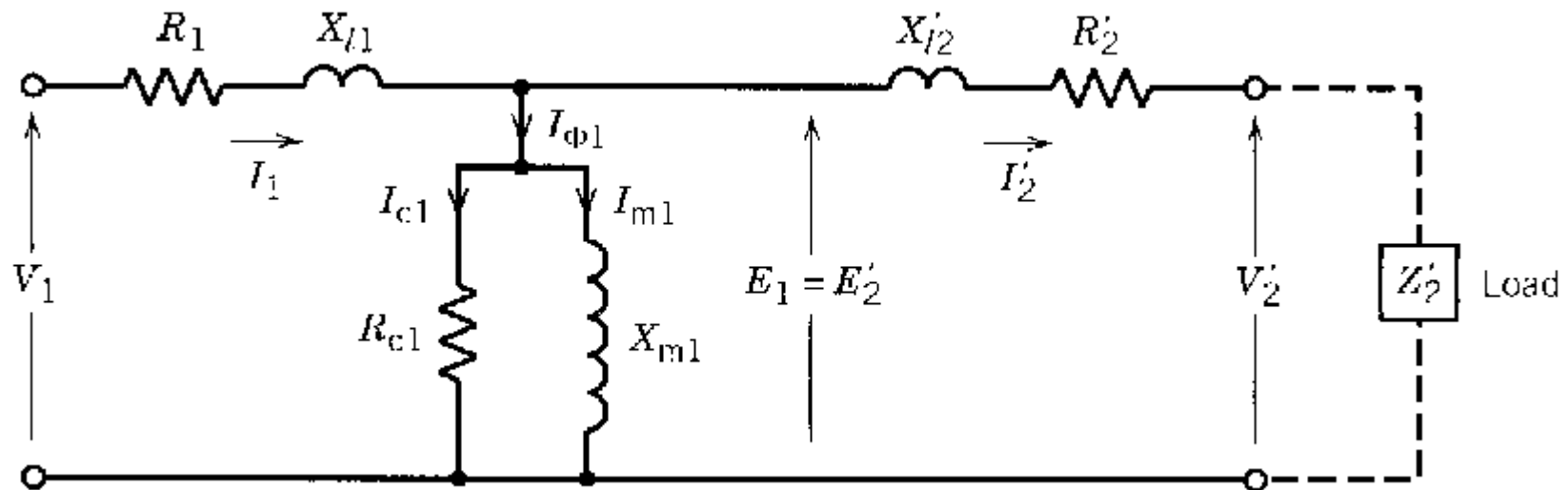
Approximate Equivalent Circuit

An approximate equivalent circuit

$$V_2' = aV_2 \quad I_2' = I_2/a$$
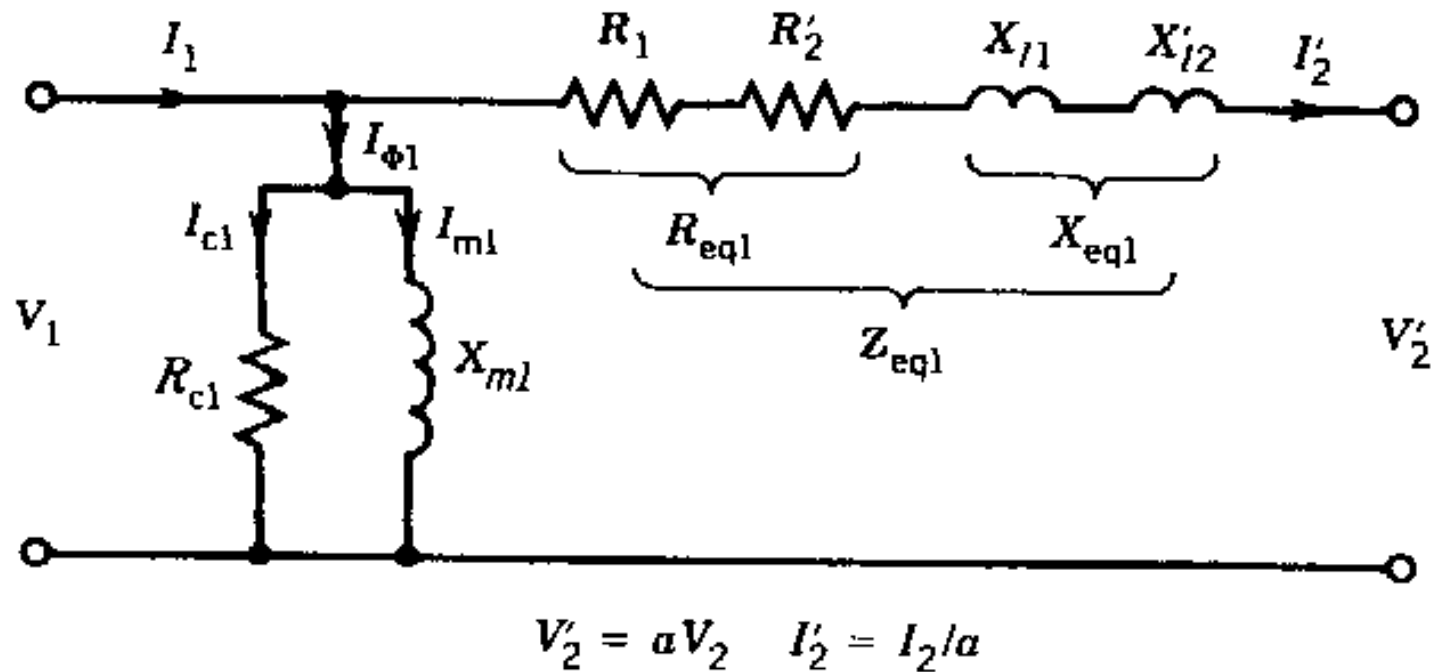
Example

A 100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3Ω and 0.01Ω respectively, and the corresponding leakage reactances are 1.1Ω and 0.035Ω respectively. Calculate the equivalent impedance referred to the primary circuit.

Re =0.55Ω, Xe = 1.975Ω, Ze = 2.05Ω

Electrical Machines 1

Hour 11

Voltage Regulation (sect. 2.3)

- Any source having internal impedance suffers a change in voltage when supplied power to a load.

- With power transformers, this is important because there is a legal limit by which the voltage at the consumer terminals may vary. (i.e. ± 6 %)

$$V.R. = \frac{\left|V_2\right|_{NL} - \left|V_2\right|_L}{\left|V_2\right|_L}$$

OR

$$V.R. = \frac{\left|V_1\right| - \left|V'_2\right|_{rated}}{\left|V'_2\right|_{rated}}$$

Example 2.3 (page 60)

Consider the transformer in Example 2.2. Determine the voltage regulation in percentage for the following load conditions.

(a) 75% full load, 0.6 power factor lagging.

(b) 75% full load, 0.6 power factor leading.

(c) Draw the phasor diagram for condition (a) and (b).

Transformer Losses

- Copper losses (I^2R losses) in primary and secondary windings

$$P_C = I_1^2 R_1 + I_2^2 R_2$$

- Iron losses (Core losses)

(a) Hysteresis loss

$$Ph = K_h B_{max}^x f V \text{ watts}$$

where K_h = a constant

f = frequency

V = Volume

B_{max} = maximum flux density

x = coefficient varies from 1.6 - 2.0 with different grades of iron.

Transformer Losses

- Iron losses (Core losses)
  (b) Eddy Current loss

  $$Pe = K_e B_{max}^2 f^2 t^2 V \text{ watts}$$

  where K_e = a constant

  f = frequency

  V = Volume

  B_{max} = maximum flux density

  t = thickness of lamination.

  Total iron losses, $P_i = P_h + P_e$

Transformer Efficiency (sect. 2.4)

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{\text{Output power}}{\text{Output power} + \text{losses}}$$

$$\eta = \frac{\text{Output VA} \times \text{power factor}}{\text{Output VA} \times \text{power factor} + \text{core loss} + I^2 R \text{ losses}}$$
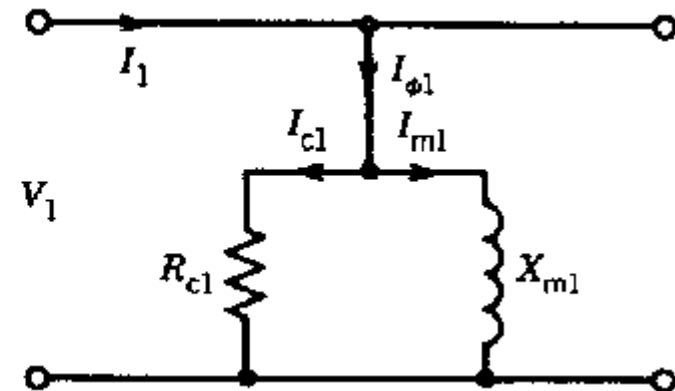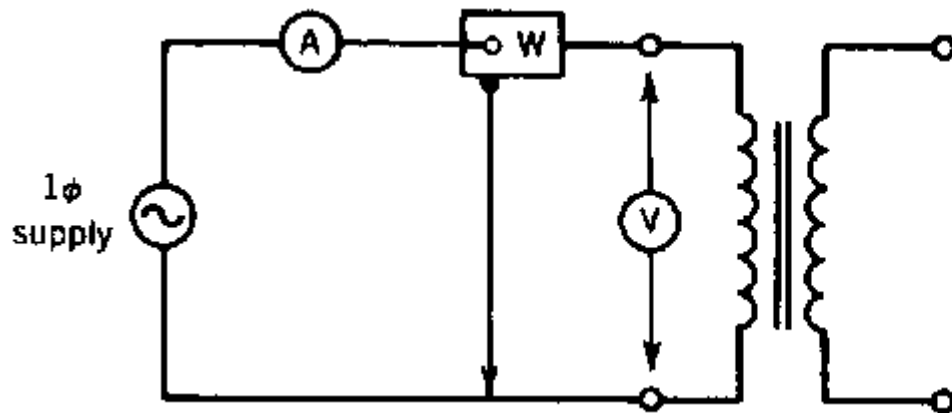
Electrical Machines 1

Hour 12

Transformer Tests (sect. 2.2.2)

Open Circuit test

- this test is performed by applying a voltage to either the HV side or LV side, whichever is convenient

- during the test, secondary winding is kept open-circuited
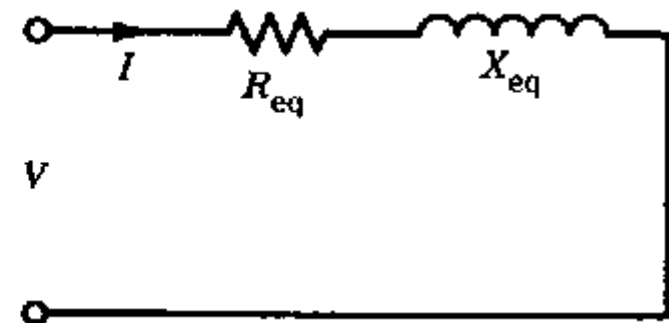
Transformer Tests

Open Circuit test (cont)

- the primary current is the exciting current and the losses measured by the wattmeter are the core losses

- core losses are essentially a fixed loss which depends on the maximum value of flux in the core and the supply frequency, but not the load

Transformer Tests

Short Circuit test

- this test is performed by short-circuiting one winding and applying rated current to the other winding

- only a small supply voltage is required to pass rated current through the windings

- it is usual to perform this test by applying a voltage to the HV side
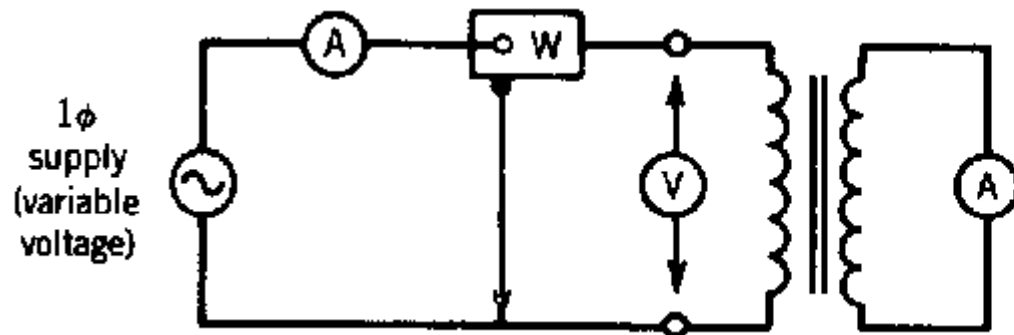
Transformer Tests

Short Circuit test (cont)

- the parameter R_{eq} and X_{eq} can be determined from the readings of voltmeter, ammeter, and wattmeter

- because the voltage applied under the short-circuit test is small, the core losses are neglected and the wattmeter reading can be taken entirely to represent the copper losses in the windings, represented by R_{eq}

Example 2.2 (page 55)

Tests are performed on a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

	Open-Circuit Test (high-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	220 V	150 V
Ammeter	2.5 A	4.55 A
Wattmeter	100 W	215 W

(a) Derive the parameters for the approximate equivalent circuits referred to the LV side and the HV side.

(b) Express the excitation current as a percentage of the rated current.

(c) Determine the power factor for the no-load and short-circuit tests.

IVE(TY)
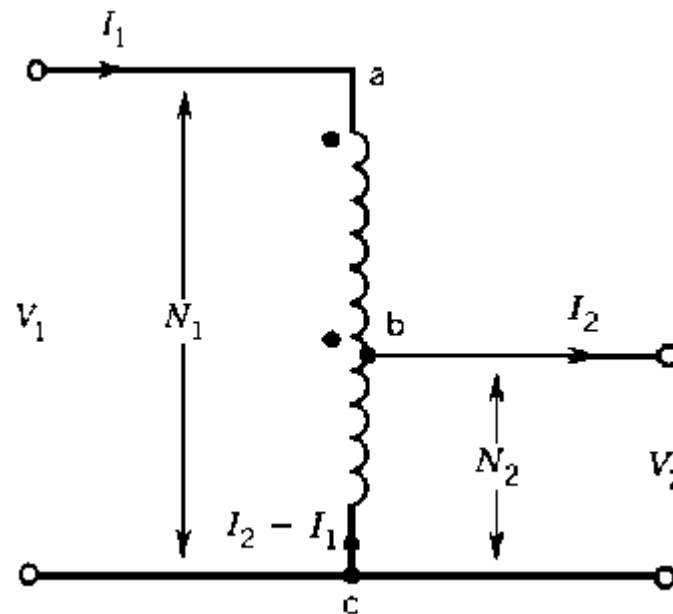*Department of Engineering*

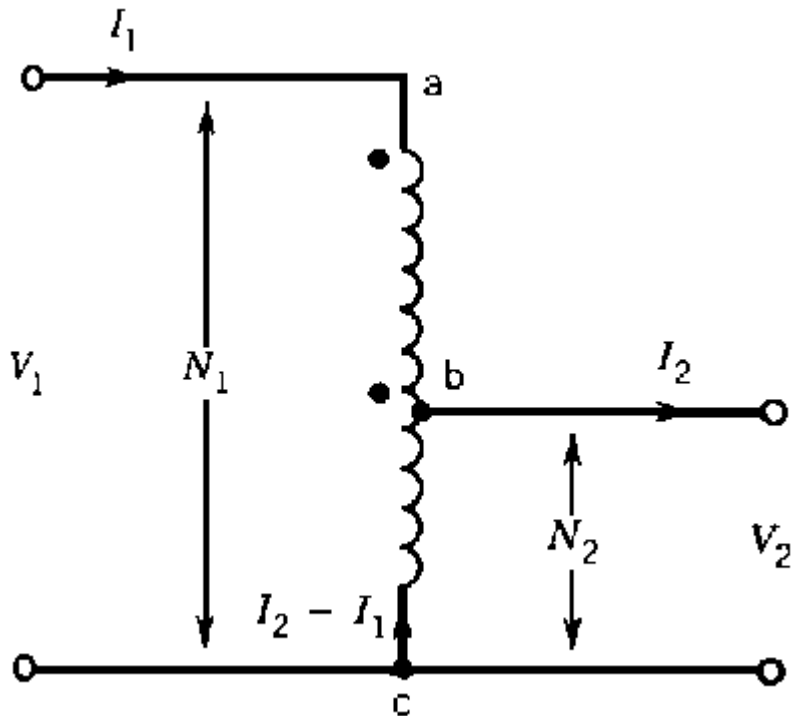# Electrical Machines 1

Hour 15

Auto-transformer (sect. 2.5)

- transformer having a part of its windings common to the primary and secondary

- when a load is connected across b and c, then a current I_2 will flow through the load. The current I_2 will produce an m.m.f. in the core which will be balanced by a current I_1 flowing in the complete winding

Auto-transformer

The voltages and currents are related by the same turns ratio as in a two-winding transformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

Example 2.6 (page 67)

A 1φ, 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig. E2.6 such that more than 2000 V is obtained at the secondary. The portion **ab** is the 200 V winding, and the portion **bc** is the 2000 V winding. Compute the kVA rating as an autotransformer.
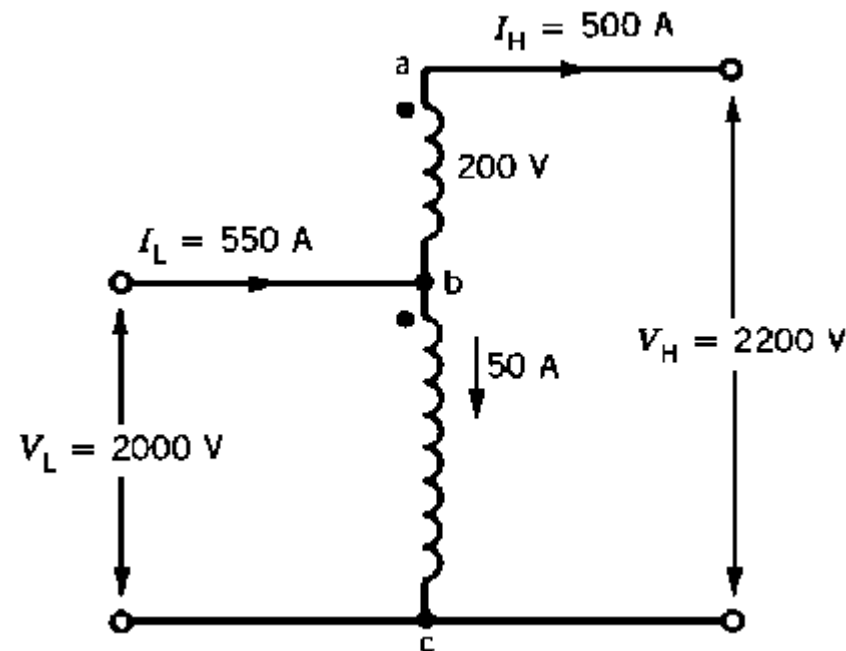
Example 2.6 (cont)

The current rating of the winding are:

$$I_{ab} = \frac{100,000}{200} = 500A$$

$$I_{bc} = \frac{100,000}{2,000} = 50A$$

Therefore, for full-load operation of the autotransformer, the terminal currents are:

$$I_H = 500A$$

$$I_L = 500 + 50 = 550A$$

Example 2.6 (cont)

Now, V_L = 2000V and V_H = 2200V

Therefore,

$$\text{kVA}\,|_{\text{L}} = \frac{2000 \times 550}{1000} = 1100 \quad \text{(ans)}$$

$$\text{kVA}\,|_{\text{H}} = \frac{2200 \times 500}{1000} = 1100 \quad \text{(ans)}$$

Note: A 1ϕ, 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA.

Advantages of Auto-transformer

- It effects a saving in winding material (copper or aluminum), since the secondary winding is part of the primary current.

- Lower copper loss, therefore efficiency is higher than in the two winding transformer.

- Lower leakage reactances, lower exciting current.

- Variable output voltage can be obtained.

Disadv. of Auto-Transformer

- There is a direct connection between the primary and secondary sides.

- Should an open-circuit develop between points **b and c**, the full mains voltage would be applied to the secondary.

- The short-circuit current is much larger than for normal two-winding transformer

Application of Auto-transformer

- Boosting or bucking of a supply voltage by a small amount. (The smaller difference voltage between the output and input voltages the greater is the saving of winding material.)

- Starting of a.c. machines, e.g. induction motor, where the voltage is raised in two or more steps from a small value to the full supply voltage.

- Continuously variable a.c. supply voltages, normally connected between a low voltage supply in and a high voltage supply out.

- Production of very high voltages, e.g. 275kV and 400kV grid system

Instrument Transformers

- Voltage Transformer VT (or PT) – N_1/N_2 is large and standard 110 V at the secondary.

- Current Transformer CT – N_1/N_2 is small and standard 5 A or 1A at the secondary.

- Reduce the voltage or current magnitudes so that instruments can be used.

- Instruments are connected to the secondary of the transformers.

- The measured values should then be multiplied by the appropriate turns ratio to get the actual primary values.

Reasons for using Instrument Transformer

- do not require special designed instruments for h.v. or heavy current measurements

- for safety reason, one terminal of the secondary winding can be earthed

- electrical isolation from the primary is achievable

About the Use of Instrument Transformer

- may have turns ratio error

- phase shift may exist between primary and secondary measurands

- CT is actually a step-up transformer with very large turns ratio (eg 500/1), excessive voltage will exist between the secondary terminals if they were left open.

- NEVER LET THE SECONDARY OF A C.T. OPEN-CIRCUITED